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Investigation of the effects of energy losses on the flow rates delivered by a siphon: a comparison of analytical and experimental results

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ABSTRACT: What was being learned in an introductory course in fluid mechanics was applied to the analysis and testing of water siphons. Different siphons had been designed, constructed and tested by different groups of students for the purpose of applying what they learned to a real-life situation. Each siphon consisted of a bucket and a beaker that were connected with a bent hose. Both containers were open to the atmosphere. Once an initial suction was introduced at the lower end of the hose, water could be lifted from the source reservoir at the higher elevation to the receiving reservoir at the lower elevation. Inviscid- and viscous-flow analyses were carried out for steady flow conditions; these were followed by experiments and all results were compared. It was found that neglecting losses overestimated the flow rates delivered by the siphon by about 370%. However, analyses that took losses into account overestimated actual flow rates by only 10%.

INTRODUCTION

As indicated by this author in an earlier article:

> *Many courses in the engineering curriculum are both mathematically intensive and challenging to students. Fluid mechanics is one of them. It is very important that students not only understand the material taught in these courses, but also that they be able to apply it to the solution of practical problems; after all, this is what most students will be doing after they graduate and go on to the world of engineering practice* [1].

There is a large body of research and classroom experience that supports the importance and the effectiveness of hands-on learning as a supplement to lectures [1]. This article gives a brief report on a hands-on project that was used in fluid mechanics lectures to achieve this purpose. This project was carried out by small groups of students who worked together. The project consists of designing, constructing, analysing and testing the performance of a water siphon made by students. The ultimate aim was to compare the predictions of analysis with the results of experiments.

A siphon is an arrangement that helps transfer liquid from one reservoir to another without the assistance of an external pump. It consists of two reservoirs of liquid that are placed next to each other but at different elevations, and a bent tube that connects the free surfaces of both reservoirs and through which the liquid in the higher reservoir is progressively transferred to the lower reservoir in a continuous manner. A sketch of a simple siphon is shown in Figure 1.

Brief, but sufficiently strong suction introduced at the lower end of the bent tube creates differences in pressure inside the tube that cause the liquid in the upper reservoir to be lifted above this reservoir and up the bent tube until it reaches its highest point on the bend. Gravity, then, pulls the liquid down from that high point to the free surface of the lower reservoir. Continuous flow through the siphon can be sustained indefinitely if, for example, one is able to maintain the level of liquid in the upper reservoir constant.

Siphoning has been used for centuries, because its mechanism has many applications [2]. For example, it is used in the flushing of household toilets, the fermentation of beer and wine to keep undesired particles away from the final container, the irrigation of cotton fields, the facilitation of the drainage of rain water from roofs, increasing the flow rates in the spillways of dams, the operation of rain gages that empty automatically and even in the circulation of blood above the human heart [2].

The exact scientific explanation of this simple mechanism is still controversial; however, in one school of thought, the siphon mechanism is driven by atmospheric pressure; in another, it is driven by liquid cohesion; in yet another, it is driven by a combination of both [2]. This article focuses on the first explanation, because that explanation can easily be proven using the application of the energy equation.

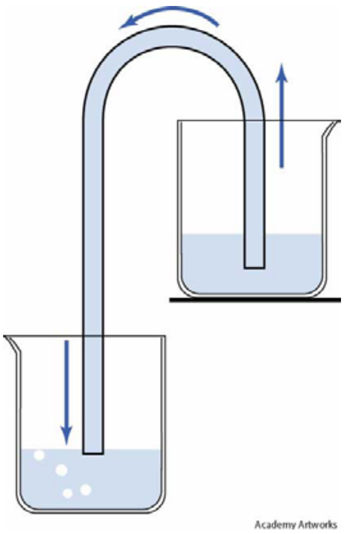

Figure1: An illustration of a simple siphon.

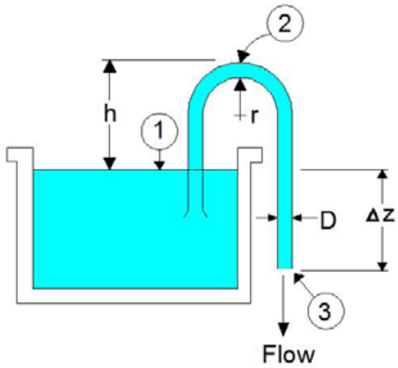

Figure 2: Sketch of a siphon with reference points shown.

The rest of the article is organised in the following manner: first the siphon mechanisms are explained using Bernoulli's equation, which assumes that flow is steady and frictionless. Second, a derivation that takes losses into account is presented. Third, a classroom exercise that is used in the first course in fluid mechanics is summarised.

THE VOLUME FLOW RATE THROUGH THE SIPHON USING INVISCID-FLOW ANALYSIS

If one applies Bernoulli's equation between points 1 and 3 for the siphon setup shown in Figure 2, one obtains:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_3}{\rho} + \frac{V_3^2}{2} + gZ_3 \quad (1)$$

Where P indicates pressure; V the average speed of the fluid through the siphon; g the acceleration of gravity; Z the elevation of a point relative to some reference level; and ρ the mass density of the fluid.

It will be assumed that the cross-section of the tank is very large compared to that of the siphon hose, thus, $V_1^2 \ll V_3^2$; and that $P_1 = P_3$, since both pressures are atmospheric. If $\Delta Z = Z_1 - Z_3$, Equation (1) leads to the approximation:

$$V_3 = \sqrt{2g\Delta Z} \quad (2)$$

If A is the cross-sectional area of the siphon hose, then, the volume flow rate, Q_i, through the siphon is:

$$Q_i = AV_3 \quad (2a)$$

Combining Equation (2) and Equation (2a), one obtains:

$$Q_i = \left(A\sqrt{2g}\right)\sqrt{\Delta Z} \quad (3)$$

Equation (3) indicates that, under steady flow conditions, the volume flow rate of inviscid fluid through the siphon is directly proportional to the square root of the length of the siphon hose that lies between the free surface of water in the source tank and the end of the siphon hose that is open to the atmosphere.

THE VOLUME FLOW RATE THROUGH THE SIPHON USING VISCOUS-FLOW ANALYSIS

If the flow is viscous, Equation (1) becomes Equation (4), given by:

$$\frac{P_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gZ_1 = \frac{P_3}{\rho} + \alpha_3\frac{V_3^2}{2} + gZ_3 + h_l \quad (4)$$

Where α is the kinetic energy coefficient and h_l represents the total head losses between points 1 and 3 [3]. If one assumes that the flow will be turbulent, then, $\alpha_1 \approx 1,\ and\ \alpha_2 \approx 1$. The losses between points 1 and 3 consist of losses at the entrance to the siphon tube, frictional losses along the wall of the siphon hose, and losses at the bend due to curvature [3]. The bend turns through an angle of 180^0, which is equivalent to two 90^0 - bends that face each other. The three loss terms added together yield what is shown in Equation (5):

$$h_l = \left[K_{ent} \ + 2f\left(\frac{L_e}{D}\right)_{90^0} \ + \ f \ \left(\frac{L}{D}\right) \ \right]\frac{\overline{V}^2}{2} \tag{5}$$

Where K_{ent} is the coefficient of losses at the entrance, $\overline{V} = V_3$, f is the Darcy-Weisbach friction factor between the wall and the flowing water, Le is the equivalent length of pipe that would produce the same energy losses as the presence of one 90^0 - bend, L is the total length of the siphon tube and D is the diameter of that tube. Standard values for these quantities, obtained from experiments [3], are: $K_{ent} = 0.78$; $\left(\frac{L_e}{D}\right)_{90^0} = 40$ and f must be obtained through iterative calculations as shown below.

If one assumes a large tank, and uses $P_1 = P_3$, as was done earlier, then, combining Equation (4) and Equation (5) and solving for $\bar{V}$ leads to:

$$\bar{V} = \sqrt{\frac{2g(\Delta Z)}{\left[1+ \ K_{ent} \ +f(\left(\frac{L}{D}\right)+ 2\left(\frac{L_e}{D}\right)_{90^0}) \ \right]}} \tag{6}$$

Using Equation (6) in Equation (2a), Q_V, the volume flow rate of viscous fluid through the siphon hose, is given by:

$$Q_V = \frac{(A\sqrt{2g})\sqrt{(\Delta Z)}}{\sqrt{\left[1+ \ K_{ent} \ +f(\left(\frac{L}{D}\right)+ 2\left(\frac{L_e}{D}\right)_{90^0}) \ \right]}} \tag{7}$$

The denominator of Equation (7) shows the effects of losses, which were neglected in the derivation of Equation (3).

Comparing Equation (3) and Equation (7), it becomes clear that one effect of losses is to reduce the flow rate below what is expected when the flow is inviscid. In order to determine how much smaller Q_V is than Q_i, one needs to calculate f, the Darcy-Weisbach friction factor.

CALCULATION OF *F* THE DARCY-WEISBACH FRICTION FACTOR

The Darcy-Weisbach friction factor *f* is a function of the relative roughness, $\frac{e}{D}$, of the tube and the Reynolds number, Re, of the flow through the tube, where e denotes the absolute roughness of the hose and D the diameter of that hose. The Reynolds number of a viscous flow with average speed V and kinematic viscosity ν through a circular tube of diameter D is given by:

$$Re = \frac{VD}{\nu}$$

It follows that the friction factor can be written symbolically as:

$$f = f(\frac{e}{D}, Re\ ) \tag{8}$$

Using the volume flow rate Q_V in the place of the velocity V, the Reynolds number can be written as:

$$Re = \frac{Q_V D}{\nu A} \tag{9}$$

The friction factor, f, is obtained using experimental data compiled in the Moody diagram shown in Figure 3 or by using the Colebrook formula shown in Equation (10). In either case, iterations are needed.

$$\frac{1}{\sqrt{f}} = -2.0\ log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{10}$$

That iteration procedure consists of four steps that are repeated as many times as necessary:

Step 1. Initiating the iteration. It is conventional to start with an initial guess for the friction factor as a way of getting the iteration process going. If $\frac{e}{D}$ is known, as is the case in this problem ($\frac{e}{D} = 0.011$), one can assume the flow to fall in the fully rough zone of the Moody diagram. In this zone, the friction factor is independent of the Reynolds number.

Hence, one can immediately obtain a starting value from Figure 3. In this case, then, using $\frac{e}{D} = \mathbf{0.011}$, one reads f = 0.04, from the Moody diagram.

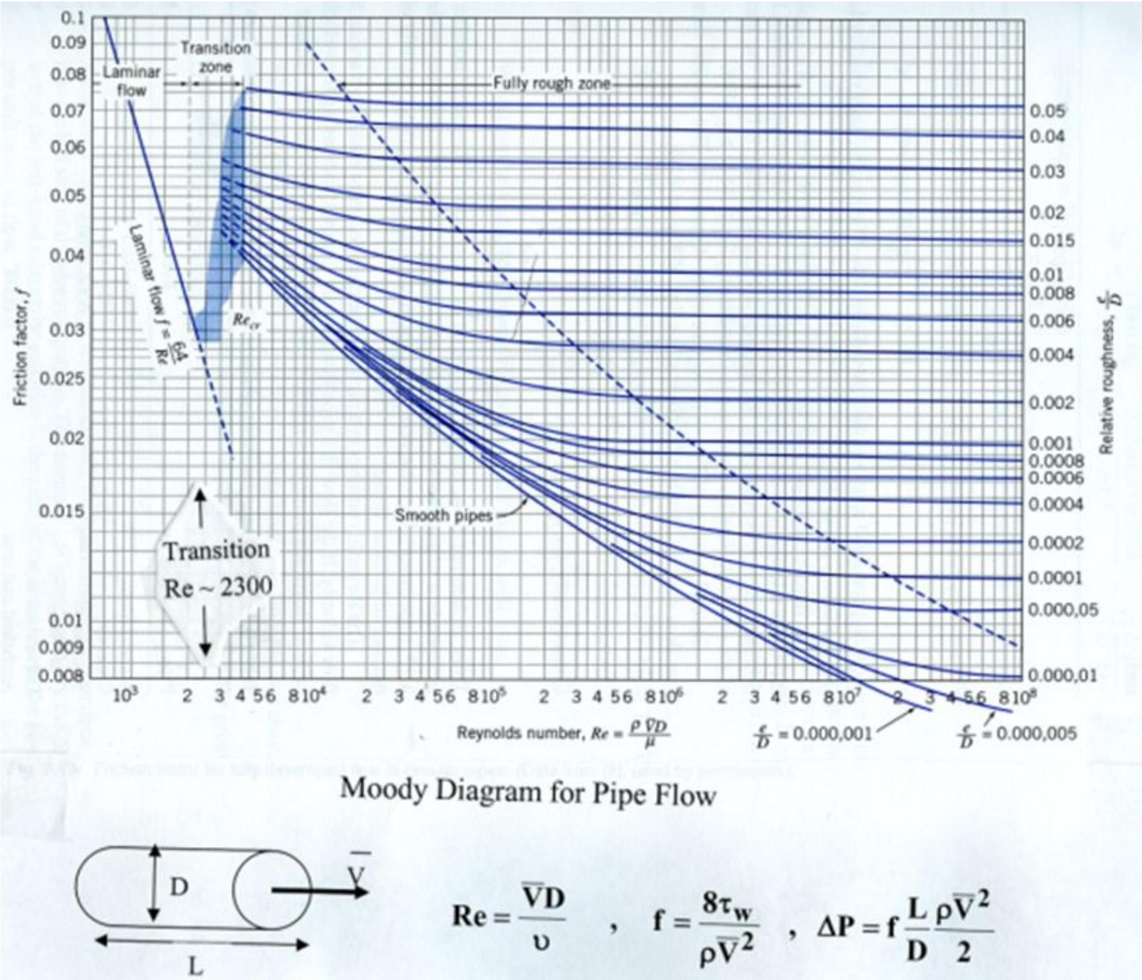

Figure 3: The Moody diagram [3].

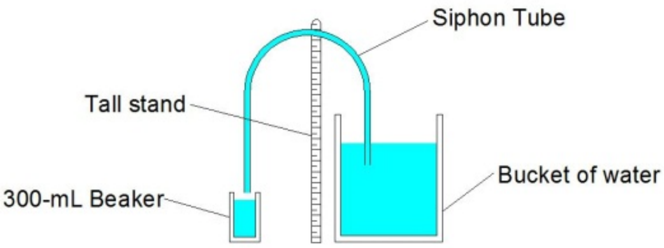

Figure 4: The experimental setup (Data used in this article were collected by study group 1: Adam Beougher, Andrew Bertsch and Jamie Coffman).

Step 2. Use the value of f from Step 1 to calculate the average volume flow rate of viscous flow using Equation (7).

Step 3. Use this value of the average volume flow rate to calculate the Reynolds number using Equation (9).

Step 4. Use this value of the Reynolds number and the relative roughness to either look up a new value of the friction factor using the Moody diagram, or to calculate it using the Colebrook formula, Equation (10). This completes the first iteration.

The second iteration. The latest value for the friction factor can be used in Step 1 to initiate the second iteration. This iterative process continues until the difference between consecutive estimates of the volume flow rate become so small that additional iterations are unnecessary.

This series of n iterations described above can be summarised in pictorial form as shown below:

$$Iteration\ 1: \left[f_0 \ \rightarrow \ (Q_V)_0 \rightarrow \left[(Re)_0, \frac{e}{D}\right] \rightarrow f_1 \right]$$
$$Iteration\ 2: \left[f_1 \ \rightarrow \ (Q_V)_1 \rightarrow \left[(Re)_1, \frac{e}{D}\right] \rightarrow f_2 \right]$$
$$Iterations\ 3\ through\ n-1: [\ldots \ldots \ldots \ldots \ldots \ldots \ldots]$$
$$Iteration\ n: \left[f_{n-1} \ \rightarrow \ (Q_V)_{n-1} \rightarrow \left[(Re)_{n-1}, \frac{e}{D}\right] \rightarrow f_n \right]$$

Figure 5: Diagram of the n iterations needed to calculate the Darcy-Weisbach friction factor.

For the siphon hose used in these experiments, the absolute roughness is e = 0.00023 ft. The curvature of the bend was found to be r = 3.75 in, D = 0.25 in, L = 5.5 ft, the kinematic viscosity of water at room temperature is $= 1.08 \times 10^{-5} \frac{ft^2}{s}$; $A = 3.409 \ \times 10^{-4} ft^2$; $\frac{e}{D} = 0.011$ (Data used in this article were collected by study group 1: Adam Beougher, Andrew Bertsch and Jamie Coffman).

THE EXPERIMENT AND ITS RESULTS

An experimental setup is illustrated in Figure 4. The materials used were 300 mL beaker, a siphon hose, a 5-gallon bucket, duct tape, a tall stand, a stop watch and water from the laboratory. The height difference, ΔZ, was varied by raising or lowering the point at which the hose was attached to the vertical stand. The volume flow rates corresponding to eight different height differences were measured and compared to those calculated using Equation (3) and Equation (7). The results are shown in Figure 6.

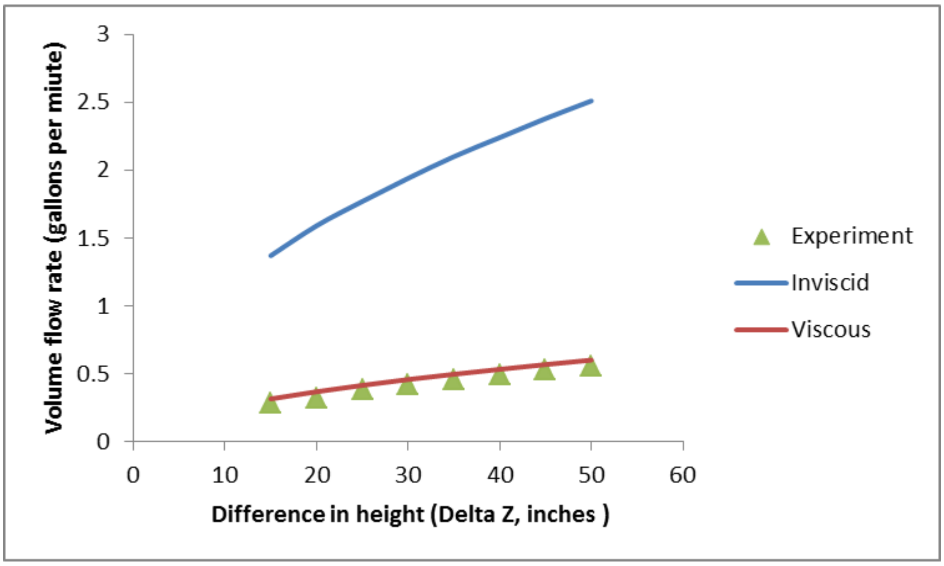

Figure 6: Graphical comparison of analytical and experimental results (Data used in this article were collected by study group 1: Adam Beougher, Andrew Bertsch and Jamie Coffman).

IMPACT ON THE LEARNING PROCESS AND IMPLICATIONS FOR ENGINEERING EDUCATION

This project had an impact on the learning process in four important ways.

It allowed students to see that, when one neglects all losses in an internal flow through a pipe, the results obtained with such an analysis are easier to get and they can be expected to predict the general trend to be followed by experimental flow rates well; but they overestimate flow rates by a very wide margin; hence, they cannot be relied upon to predict the magnitudes of the actual flow rates to be expected. However, when one includes all losses that can be reasonably accounted for, the required analysis is more complicated; but results so obtained not only predict the general trend well, they also yield magnitudes of volume flow rates that are much closer to those measured in the laboratory [4].

It gave students who learn by doing an opportunity to use their preferred learning style.

The fact that students were divided into small groups that worked together created supportive micro communities within the class. Students interacted with each other, assisted each other and supported each other within those communities. As a consequence, the persistence rate in the course increased. Indeed, no student withdrew from the course; this included those who ended up failing the course.

And, students were very engaged with the material that was being learned; engagement with the material was gauged by the amount of time and energy students spent on carrying out the tasks that required a lot of effort and creativity.

The strength of that engagement translated itself into higher academic performance: scores earned in exercises involving hands-on projects were higher than those earned in similar in-class examinations and quizzes over other course topics. Thus, whereas the average scores earned by students in in-class assessment exercises were approximately 70%, those earned in hands-on projects averaged 88%, an increase of 18%. If one assumes, as is commonly done, that such scores are indicative of what students learned, then, hands-on projects led to greater learning and comprehension [5].

The project also has implications for engineering and technology education in three important ways.

It is very expensive to equip and maintain an engineering laboratory by buying everything that is needed. It is possible to reduce the cost of running a given laboratory by supplementing the equipment that exists in that laboratory with home-made units that are constructed and tested for specific experiments and demonstrations by the students themselves. This project and others that have been conducted in the author's laboratory demonstrate how that can be done [1].

One can use textbook exercises as a springboard for this process. A textbook exercise can be solved analytically first, then, it can be implemented in the laboratory for the purposes of testing and verification of analytical results. In this project, for example, the siphon was a textbook exercise that was solved analytically; first by assuming an ideal operation that neglected losses; then, by using a more realistic analysis that included the effects of losses. Implementation through an experiment allowed students to design, build and test a model of the textbook exercise by themselves and, subsequently, compare the experimental results to those obtained analytically. The end result was that students were able to assess the extent to which the results of their analyses were realistic [3].

Whereas hands-on learning has been demonstrated to be very effective in elementary and secondary education, its inclusion and acceptance in lecture courses in colleges and universities has been slow [6]. This situation persists despite the considerable evidence that exists in the literature in support of the benefits of hands-on learning [1][5]. Indeed, Chickering and Gamson [7] have summarised the evidence that supports hands-on learning into seven effective practices in undergraduate teaching and learning; a project, such as the one on the siphon utilised six of those practices. They are: student-staff contact, active learning, time on task, high expectations, respect for diverse learning styles, and cooperation amongst students. Prompt feedback, the seventh effective practice, could not be utilised, because the project described herein was assigned at the beginning of the semester and was not due until the very end of the semester.

Furthermore, many studies have shown that engaging students in the learning process, as is done in hands-on projects, has positive correlations with the improvement of a variety of outcomes that are desirable in undergraduate education. Among these are persistence in course work [8], increases in confidence and in the ability to transfer what students learned in one context to another [9], enhancements in cognitive development [10] and improvements in academic performance that manifest themselves in the form of better grades [11].

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